Application of a Volume Averaged $k$-$\varepsilon$ Model to Particle-Laden Turbulent Channel Flow

A closed set of volume averaged equations for modeling turbulence in the carrier phase of particle-laden flows is presented. The equations incorporate a recently developed dissipation transport equation that contains an additional production of dissipation term due to particle surfaces. In the development, it was assumed that each coefficient was the sum of the coefficient for single phase flow and a coefficient quantifying the contribution of the particulate phase. To assess the effects of this additional production term, a numerical model was developed and applied to particles falling in a channel of downward turbulent air flow. Boundary conditions were developed to ensure that the production of turbulent kinetic energy due to mean velocity gradients and particle surfaces balanced with the turbulent dissipation near the wall. The coefficients associated with the production of dissipation due to mean velocity gradients and particle surfaces were varied to assess the effects of the dispersed phase on the carrier phase turbulent kinetic energy across the channel. The results show that the model predicts a decrease in turbulent kinetic energy near the wall with increased particle loading, and that the dissipation coefficients play a critical role in predicting the turbulent kinetic energy in particle-laden turbulent flows. [DOI: 10.1115/1.3203204]

Keywords: turbulence, modeling, particles, dissipation, volume average

1 Introduction

Particle-laden turbulent flows are common in industrial applications. Large industries such as the chemical, pharmaceutical, agricultural, and mining industries can benefit from an understanding of particle-laden turbulent flows [1]. Two-phase gas-solid suspension flows are prevalent in chemical engineering applications such as spray drying, cyclone separation, pneumatic conveying, pulsed coal gasification, and combustion [2]. Turbulence is also responsible for enhancing heat transfer, mixing of chemical species, and altering the wall shear stress [3].

The addition of particles to a turbulent flow has been experimentally shown to alter the carrier phase turbulence [4–8]. Particle-laden turbulent flows are complicated to model, and understanding the physics well enough to develop a general model to predict the interactions between the continuous and dispersed phases becomes even more of a challenge. The ideal approach to modeling these interactions would require boundary conditions at the surface of each particle in the flow. However, the mesh associated with this type of analysis would require enormous computational efforts. In order to circumvent the computational difficulties associated with considering every particle, many researchers [2,9–14] have assumed that the effect of the particles could be incorporated by including a force to the instantaneous form (i.e., a point in space and an instance in time) of the momentum equation, such as

$$
\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) + \rho g_i + F_i
$$

where $F_i$ is a force per unit volume due to the presence of particles. In order for the above equation to be valid at a point (e.g., the limiting volume of the fluid [15]), the force due to the particles must be continuous and definable. However, the term $F_i$ is not definable at any point in the continuous phase. To better understand this claim, consider an arbitrary particle and an arbitrary point near the particle, as shown in Fig. 1(a). The effect of the particle on the fluid within some arbitrary control volume cannot be represented by a force per unit volume, $F_i$.

The force on the fluid due to the particles can only be defined if the control volume contains enough particles, as shown in Fig. 1(b). In order for $F_i$ to be valid at a point, it must be continuous; thus the limiting volume would have to contain sufficient particles to have stationary average properties. For cases which typify fluid-particle flows this limiting volume would be many orders of magnitude larger than a point in the conveying phase. In addition, a pointwise approach to modeling turbulence in particle-laden flows by decomposing the force, $F_i$, into mean and fluctuating components, in time, is also meaningless based on the above argument. Several turbulence models [2,9–14,16] have been developed for particle-laden flows, most of which are based on temporal averaging approaches. A fundamental approach to capture the effects of the particles in the momentum, turbulent kinetic energy (TKE), and dissipation equations is through volume averaging (a further discussion is presented in Sec. 2).

To assess the validity of volume averaging, Zhang and Reese [17] studied the additional source and sink terms due to the presence of particles in the TKE equations proposed by Crowe and Gillant [18] and Chen and Wood [2] and compared them to the data of Tsuji et al. [4]. They concluded that the temporal averaged model did not predict the altered turbulent intensity due to particle mass loading, but they showed that the volume averaged model did produce changes in turbulent kinetic energy with respect to particle loading. Overall, they conclude that the generation terms proposed by Crowe and Gillant [18] more accurately match the experimental data.

In a recent review of the status of modeling particle-laden turbulence, Eaton [19] stated that many models have been developed to understand the turbulence modulation associated with dilute particle-laden flows, yet there still remains a general model that can account for subjective factors such as particle size, relative Reynolds number, volume fraction, number density, mass density,
surface roughness, etc. The present study reviews a fundamental approach to obtaining a system of equations that include the surface effects of particles and many of the subjective factors identified by Eaton [19] and Curtis and van Wachem [1]. In addition, the modeling procedure is further developed and applied to the experimental data of Kullick et al. [6] and Paris and Eaton [8]. Furthermore, the production coefficients within the dissipation equation are varied to assess their effect on the turbulent kinetic energy.

2 A Review of the Volume Averaged Equation Set for the Continuous Phase Turbulence in Particle-Laden Flows

The volume average momentum equations can be obtained from Newton’s second law [20] or by volume averaging the Navier–Stokes equations [21]. When particle rotation and mass transfer are neglected, both methods show that the volume averaged momentum equations for incompressible, particle-laden flows are of the form

$$\frac{\partial}{\partial t} \langle u_i \rangle + \frac{\partial}{\partial x_j} \alpha_c \langle \rho u_i u_j \rangle = -\alpha_c \frac{\partial \langle P \rangle}{\partial x_j} + \alpha_c \frac{\partial \langle \tau_{ij} \rangle}{\partial x_j} + \alpha_c \langle \rho g_i \rangle$$

$$- \frac{3 \pi \mu}{V} \sum_n f D(u_i - v_i)$$

where \(\alpha_c\) is the volume fraction of the continuous phase, \(\rho\) is the density of the continuous phase, \(V\) is the mixture volume, \(f\) is the drag factor, \(D\) is the particle diameter, \(n\) is the particle number, and \(\langle \cdot \rangle\) is the volume average [21]. For the above equation to be valid, a control volume, larger than the limiting volume of the fluid that contains \(N\) number of particles, must be used to obtain a statistical average, illustrated in Fig. 2. Some researchers [9] have used Reynolds averaging procedures on the volume averaged equations. However, the average velocities in the volume averaged equations do not represent the local (pointwise) instantaneous velocity of a given flow and thereby are not amenable to the Reynolds averaging procedures used in single phase flows. In other words, the temporal fluctuations of the averaged velocities do not reflect the flow turbulence of the carrier phase.

Aside from temporal decomposition, another way of identifying turbulence within the carrier phase is by the velocity deviation from the volume averaged velocity, such as [18]

$$u_i = \langle u_i \rangle + \delta u_i$$

where \(u_i\) is the instantaneous velocity (i.e., the velocity at an instance in time and at an arbitrary location within the continuous phase region of the volume of interest), \(\langle u_i \rangle\) is the volume average velocity, and \(\delta u_i\) is the volume deviation velocity, as illustrated in Fig. 3. Here the instantaneous velocity is used to describe the velocity at a point within the continuous phase of the control volume, which allows one to capture the local effects cause by turbulence.

The volume averaged momentum equations for particle-laden turbulent flows are obtained by substituting Eq. (3) into Eq. (2) and applying volume averaging techniques. This decomposition presents a volume deviation stress, which is analogous to the Reynolds stress found by Reynolds averaging procedures. Assuming that the material properties of the continuous phase are constant over the volume of interest, Eq. (2) becomes

$$\alpha_c \frac{\partial \langle u_i \rangle}{\partial t} + \alpha_c \frac{\partial \langle u_i u_j \rangle}{\partial x_j} = -\frac{\alpha_c}{\rho} \frac{\partial \langle P \rangle}{\partial x_j} - \frac{\partial}{\partial x_j} (\alpha_c \langle \delta u_i \delta u_j \rangle) + \alpha_c \langle g_i \rangle$$

$$+ \alpha_c \frac{1}{\rho} \frac{\partial \langle \tau_{ij} \rangle}{\partial x_j} - \frac{3 \pi \mu}{V} \sum_n f D(\langle u_i \rangle - \delta u_i - v_i)$$

The above equation describes the momentum of the continuous phase in particle-laden turbulent flows, but it requires a closure model to evaluate the volume deviation stress.

To close the equation set, the turbulent-viscosity hypothesis is assumed to apply. The volume deviation stress is then modeled as

$$\langle \delta u_i \delta u_j \rangle = \frac{2}{3} \delta_{ij} k - v_T \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right)$$

where \(v_T\) is the turbulence viscosity defined as

$$v_T = C_w k^2 \frac{\nu}{\varepsilon}$$

where \(k\) is the volume average turbulent kinetic energy defined as [18]

$$k = \frac{1}{V_c} \int_{V_c} \frac{\delta u_i \delta u_i}{2} dV$$

and \(\varepsilon\) is the volume average dissipation defined as [18]

$$\varepsilon = \frac{1}{V_c} \int_{V_c} \frac{\partial \delta u_i \partial \delta u_i}{\partial x_j} dV = \nu \left( \frac{\partial \delta u_i \delta u_i}{\partial x_j} \frac{\partial \delta u_j \delta u_j}{\partial x_i} \right)$$

and \(C_w\) is a constant (0.09). Thus a transport equation for the volume average turbulent kinetic energy and dissipation is needed.

A volume average turbulent kinetic energy transport equation was derived by Crowe and Gilliland [18]. The transport equation for turbulent kinetic energy within the carrier phase of a particle-laden flow is shown to be [18]
\[
\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \mathbf{u} + \frac{\partial}{\partial x_i} \left( \rho u_i u_j \right) + \frac{\partial}{\partial x_j} \left( \rho u_j u_i \right) - \frac{\rho}{\mathcal{V}} \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} \right) - \rho \mathbf{e} \cdot \nabla \mathbf{e}
\]

where \( \mathbf{e} \) is the hydraulic drag coefficient defined as [21]

\[
\mathbf{e} = \frac{\alpha_d \rho u_d f}{\tau_p}
\]

and \( \alpha_d \) is the volume fraction of the dispersed phase, \( \rho_d \) is the material density of the dispersed phase particles, and \( \tau_p \) is the particle response time.

A volume average turbulent dissipation transport equation was derived by Schwarzkopf et al. [22]. The form of the transport equation for turbulent dissipation is found to be

\[
\frac{\partial (\rho_d u_i)}{\partial t} = -\rho_d \nabla \cdot \left( \rho_d u_i u_j \right) + \frac{\partial}{\partial x_j} \left( \rho_d u_j u_i \right) - \frac{\rho_d}{\mathcal{V}} \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j} \right) - \rho_d \mathbf{e} \cdot \nabla \mathbf{e}
\]

with the assumptions of incompressible flow, irrotational particles, and no mass transfer. The above equation contains three integrals that represent the effects of particles on the turbulence within the carrier phase. The terms representing the effects of the particles were evaluated by assuming that the particles were smaller than the equivalent Kolmogorov length scale, such that the Stokes drag law would apply [22]. Based on this assumption, the development revealed a production term due to the presence of particles. The form of this term was then modified to extend beyond Stokes drag. In keeping with the modeling procedures used with single phase flows, the dissipation Eq. (11) becomes [22]

\[
\frac{\partial (\rho_d u_i)}{\partial t} = -\alpha_d C_{k1} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) + C_{k2} \rho \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} - C_{k2} \rho \mathbf{e} \cdot \nabla \mathbf{e}
\]

where \( C_{k1} \) is the coefficient associated with the production of dissipation due to mean velocity gradients, \( C_{k2} \) is the coefficient associated with dissipation due to particle surfaces. The coefficients \( C_{k1} \) and \( C_{k2} \), shown in the above equation, are assumed to be a linear sum of the single phase coefficient and a coefficient that captures the effect of the particles. The ratio of the \( C_{k1}/C_{k2} \) was found by reducing data from experiments involving turbulence generation by particles in homogeneous flows [22] and can be represented by

\[
\frac{C_{k1}}{C_{k2}} = C_{e_p} \cdot \text{Re}_{e_p}^m
\]

where \( C_{e_p} \) is the fit coefficient equal to 0.058, \( m \) is equal to 1.416, and \( \text{Re}_{e_p} \) is the Reynolds number based on the relative velocity between the particles and the fluid. The values of \( C_{e_p} \) and \( m \) represent data for \( \text{Re}_{e_p} \geq 100 \), and it is hypothesized that the particle loading has an additional effect at lower \( \text{Re}_{e_p} \). The coefficient for the dissipation of dissipation \( (C_{k1}/C_{k2}) \) was found from a direct numerical simulation (DNS) study of isotropic turbulent decay with stationary particles [23] and can be represented by

\[
C_{e_p} = C_{e_p} + 0.355 \cdot \text{Re}_{e_p}^{0.99}
\]

where \( C_{e_p} \) is the standard coefficient for single phase flow (1.92) and \( \text{Re}_{e_p} \) is the dimensionless momentum coupling factor [23]. It is noteworthy to mention that the dimensionless momentum coupling factor is related to the ratio of the dispersed phase mass concentration to the Stokes number \( \bar{n} \). The production of dissipation coefficient due to mean velocity gradients \( (C_{k1}) \) is unknown but assumed to have the following functional form [20]:

\[
C_{k1} = C_{e_p} + \text{func}(\text{C}, \text{St}, \text{Re}_{e_p}, \text{Re})
\]

where \( \text{Re}_{e_p} \) is the particle Reynolds number, \( \text{Re} \) is the fluid Reynolds number, and \( C_{e_p} \) is the standard coefficient for single phase flow (1.44). The coefficient \( C_{k1} \) can be solved for analytically [20], however, data needed to evaluate the above coefficient are lacking. The effect of this coefficient is studied in Sec. 4.

### 3 Modeling the Continuous Phase Turbulence in Particle-Laden Channel Flow

To validate the volume average equation set presented above, we consider the data of Kulick et al. [6] and Paris and Eaton [8]. Graham [24] claimed that these data sets are challenging to predict with simple models and a more sophisticated model is needed. Eaton [10] claimed that a new model for the dissipation rate is needed to represent the additional dissipation generated by particles. The above dissipation rate Eq. (12) includes a production of dissipation due to particles.

Kulick et al. [6] and Paris and Eaton [8] used the same experimental setup, which was a high aspect ratio vertical channel with a development section of 5.2 m. The particles fell down the channel in the same direction that the air was traveling. The particle response time for the 70 mm copper particles is approximately 130 ms [6]. Therefore these particles should have reached a terminal velocity well before the measurement section, and the particle velocity should have exceeded the air velocity throughout the channel. However, the data sets show that the air velocity exceeds the particle velocity at the center of the channel, yet the particle velocity exceeds the air velocity near the wall. The volume fractions of the particles are on the order of 10⁻³, which implies the importance of two-way coupling and allows one to neglect particle-particle interaction [25]. However, the work of Nasr and Ahmadi [13] showed that when particle-particle and particle-wall collisions are included, a nearly flat particle velocity profile is obtained.

The data presented by Kulick et al. [6] and Paris and Eaton [8] are for steady fully developed particle-laden turbulent channel flow. For such a flow, a simple 1D modeling approach can be taken. Additional assumptions are uniform particle diameter, no mass transfer between the dispersed and continuous phase, particle rotational effects are neglected, and the particle velocity deviations are negligible compared with the mean slip velocity. The volume averaged conservation equations (4), (9), and (12) for the continuous phase are applied to a steady incompressible fully developed flow and are shown as follows.

For continuous phase momentum,

\[
0 = -\alpha_d \frac{d(P)}{dx} + \alpha_d \rho_g + \alpha_d \frac{d}{dy} \left( \mu + \mu_f \right) \frac{d(u)}{dy} - \beta_s(u - \langle u \rangle)
\]

For continuous phase turbulent kinetic energy,
The above equation assures that the net production of turbulent kinetic energy due to particles and mean velocity gradients is balanced by the dissipation and the diffusion is assumed negligible. Applying these assumptions to Eq. (17) shows

\[ 0 = \alpha_\tau \left( \frac{d(u)}{dy} \right)^2 + \frac{\beta_v}{\rho_c} [(u) - \langle u \rangle]^2 + \alpha_\tau \frac{d}{dy} \left( \nu + \frac{\nu_f}{\sigma_e} \right) \frac{dk}{dy} - \alpha_\tau e \]  

(17)

For continuous phase turbulent dissipation,

\[ 0 = C_1 \alpha_\tau \frac{d(u)}{dy} + E_k^2 \frac{\alpha_\tau C_e}{k} - \alpha_\tau \frac{d}{dy}\left( \nu + \frac{\nu_f}{\sigma_e} \right) + \alpha_\tau \frac{d}{dy}\left( \nu + \frac{\nu_f}{\sigma_e} \right) \frac{dk}{dy} - \alpha_\tau e \]  

(18)

where the turbulence kinematic viscosity \( \nu_f \) is modeled by Eq. (6) and the coefficients, shown in the above equations, are given in Table 1. At the wall, the velocity of the continuous phase is zero and the shear at the wall is modeled by

\[ \tau_w = \frac{\rho_c C_1^{1/2} \nu}{u^*} u \]  

(19)

Near the wall, the production of turbulent kinetic energy is balanced by the dissipation and the diffusion is assumed negligible. Applying these assumptions to Eq. (17) shows

\[ e = \nu_1 \left( \frac{d(u)}{dy} \right)^2 + \frac{\beta_v}{\rho_c} [(u) - \langle u \rangle]^2 \]  

(20)

The above equation assures that the net production of turbulent kinetic energy due to particles and mean velocity gradients is balanced by dissipation near the wall. Based on the data of Kulick et al. [6] and Paris and Eaton [8], it was assumed that the law of the wall can be applied to particle-laden flows; therefore, the dissipation near the wall is shown to be of the form

\[ e = \frac{u_*^2}{k^2} \frac{\beta_v}{\rho_c} [(u) - \langle u \rangle]^2 \]  

(21)

The turbulent kinetic energy near the wall is found by assuming that the shear stress is constant near the wall, and the shear at the wall is related to the friction velocity by

\[ u_*^2 = C_e \frac{k^2}{\epsilon} \frac{d(u)}{dy} \]  

(22)

Again, assuming the law of the wall is valid and eliminating dissipation from Eqs. (21) and (22) yields the following relation for the particle-laden turbulent kinetic energy near the wall:

\[ k = \frac{u_*^2}{\sqrt{C_e}} \sqrt{\left( 1 + \frac{\beta_v k^2}{\epsilon} [(u) - \langle u \rangle]^2 \right)} \]  

(23)

A finite volume approach [26] was used to model Eqs. (16)–(18), and the boundary conditions applied near the wall were Eqs. (19), (21), and (23). The particle velocity profile was determined from the experimental data [6,8]. The turbulent kinetic energy could not be directly found from the data. For purposes of analysis, it was assumed that the turbulence was homogeneous in the transverse directions. The turbulent kinetic energy for laden and unladen cases was obtained from

\[ 2k = \overline{u'^2} + 2 \cdot \overline{u'^2} \]  

(24)

For the unladen case, the standard coefficients were used in the \( k-e \) model; for the laden cases, the coefficients were the sum of the single phase coefficients and coefficients to account for the effects of the dispersed phase (see Table 1). The unladen piezometric pressure gradient was determined from the data of Kulick et al. [6] and found to be \(-14.28 \) Pa/m. Kulick et al. [6] could not measure the laden pressure gradient; Paris and Eaton [8] measured the laden pressure gradient but could not justify why it was so high. In the experiments of Kulick et al. [6] and Paris and Eaton [8], the continuous phase mass flow rate was adjusted for the different particle loadings in order to maintain a constant centerline velocity, shown in Table 2. In the model, for the laden cases, the piezometric pressure gradient was iterated until the centerline velocity matched that of the experimental data.

4 Results and Discussion

The unladen velocity profile and turbulent kinetic energy predicted by the model was compared with the data of Kulick et al. [6] and Paris and Eaton [8]. For this case, the standard \( k-e \) model was used with the standard coefficients (shown in Table 1). The velocity was normalized by the centerline velocity and plotted in terms of \( Y^+ \). The normalized unladen velocity profile predicted by the model agrees well with the measurements, shown in Fig. 4. Kulick et al. [6] used laser Doppler anemometry (LDA), and Paris and Eaton [8] used particle image velocimetry (PIV) to measure the mean and fluctuating velocities. The turbulent kinetic energy was determined from the data using Eq. (24); the data agree well with the model and are shown in Fig. 5.

Kulick et al. [6] obtained air velocity measurements for various particle sizes (50–90 \( \mu m \)) and loadings (0–0.8). The data col-

![Fig. 4 Comparison of the unladen velocity profiles predicted by the model to the experimental data of Kulick et al. [6] and Paris and Eaton [8]](image-url)
lected by Kulick et al. [6] demonstrate that 50 μm and 90 μm glass particles produce little attenuation, however, 70 μm copper particles at mass loadings greater than 0.1 produce significant turbulence attenuation. For the case of 70 μm copper particles at mass loadings of 10% and 20%, the experimental data of Kulick et al. [6] are compared with the model predictions. For the particle-laden case, the coefficients used in the volume averaged k-ε equations are shown in Table 1. The velocity is normalized by the centerline velocity and shown in Fig. 6. For the 10% and 20% mass loading cases, the model shows to underpredict the velocity magnitude near the wall. It is also noticed that the velocity profiles predicted by the model are quite different than those found from the experimental data. The velocity profile predicted by the model for a mass loading of 10% shows a slight increase in the carrier velocity near the center of the channel, while the increase in velocity for a mass loading of 20% is more pronounced for y/h > 0.6. The turbulent kinetic energy predicted by the model is compared with experimental data for 10% mass loading (Fig. 7) and 20% mass loading (Fig. 8). The model compares well with the experimental data near the wall, but deviates toward the center of the channel. For the 20% mass loading case, a flat profile is noticed near the center of the channel. Although not seen in the data set of Kulick et al. [6], a similar flat profile for the turbulent kinetic energy near the centerline of a pipe is found in the data of Tsuji et al. [4] and Sheen et al. [5] for small particles. To illustrate the capability of the model predicting a decrease in TKE with increased mass loading, a comparison of the two mass loadings is shown in Fig. 9. Near the wall, the model predicts turbulence attenuation for increased mass loading, in agreement with the experimental data. Near the centerline of the channel, the predictions deviate from the experimental data. Paris and Eaton [8] used the same experimental set up by Kulick et al. [6] and were able to reproduce the unladen flow characteristics. The data of Paris and Eaton [8] stops short of the centerline of the channel; a reason is not given. The particle size and loading is shown in Table 2. A comparison of the velocity profile normalized by the centerline velocity for the laden case of 20% mass loading is shown in Fig. 10. The magnitude of the velocity profile is slightly lower than the experimental values but the predicted velocity trend agrees with the data. It is noteworthy to mention that the normalized velocity profile reported by Paris and Eaton [8] for the continuous phase matches very well to the unladen velocity profile of Kulick et al. [6], yet the model predicts two different velocity profiles. In light of the model, the only difference between these two data sets is the particle properties and the particle velocity profile, the last of which was determined.
by different measurement techniques. The predicted turbulent kinetic energy is compared with the measurements and shown in Fig. 11.

Part of the reason for these deviations may be explained by the noninclusion of the redistribution terms in the turbulence energy equation, Eq. (9). For most cases involving particle-laden flows, \((\overline{u'v'})^2 \gg \langle \partial \overline{u} \partial \overline{v} \rangle \) so the redistribution terms may be neglected for such cases. However, for cases when \((\overline{u'v'})^2 \approx 0\), the redistribution terms may play a significant role in modeling turbulent kinetic energy near the wall. In the data of Kulick et al. [6] and Paris and Eaton [8], there is a point in the flow where the continuous phase and dispersed phase velocities are equal, but the data set does not give correlations for \(\langle \partial \overline{u} \partial \overline{v} \rangle\). A method to model these terms has been proposed by Zhang and Reese [16] and may need to be considered.

Another reason for the deviation of turbulent kinetic energy near the center of the channel is the fact that the coefficient \(C_{\varepsilon3}\) was calibrated with minimal data at low relative Reynolds numbers [22]. The data at low relative Reynolds numbers suggested that particle loading affects the coefficient. To evaluate the effectiveness of the additional production term in the dissipation model, the production coefficient \(C_{\varepsilon p}\) is varied. Within this coefficient, \(C_{\varepsilon p}\) is varied from zero to 0.060 (to simulate an effect of particle concentration), while the exponent \(m\) remained at 1.416 (the effect of particle Reynolds number). When \(C_{\varepsilon p}\) is set to zero, the additional production of dissipation due to particle surfaces is suppressed, yet the effects of the particles are still included in the turbulent kinetic energy equation. For such a case the dissipation is essentially modeled as the single phase dissipation with the standard coefficients (the effect of particles on \(C_{\varepsilon2}\) is negligible). This formulation shows a nearly constant TKE across the channel (see Fig. 12). Increasing \(C_{\varepsilon p}\) to 0.015 shows a drastic reduction in TKE. For this case, the TKE profile is nearly matched, but the predicted magnitude is higher than the experimental data. As \(C_{\varepsilon p}\) is increased to 0.060, the TKE near the wall matches well; near the center of the channel, the model underpredicts the measurements and shows a constant TKE. The normalized velocity profiles show that as the production coefficient is reduced from 0.060 to 0.015, the trend and magnitude of the velocity profiles better match the data (shown in Fig. 13). It can be seen that if the production coefficient was zero, the velocity profile would be overpredicted; this substantiates that a production of dissipation term due to particles is necessary but a better calibration of the coefficient is needed at lower particle Reynolds numbers.

The other unknown is the production coefficient \(C_{\varepsilon p}\) due to mean velocity gradients. In the derivation of the dissipation equation, the production coefficient was assumed to vary with one or
more of the fundamental nondimensional parameters found in particle-laden flows, namely, particle loading, Stokes number, particle Reynolds number, or the Reynolds number of the flow [20]. The calibration of this coefficient is complex [20] and depends on the shear velocity and von Karman constant, both of which are unknown in the data presented in the literature. To simplify, \( C_{\varepsilon 1} \) was assumed constant between the wall and the center of the channel yet altered from its standard value \((1.44)\) to understand its effect (see Fig. 14). In this figure, the coefficient \( C_{\varepsilon 1} \) is set to 0.023, which matches the magnitude of TKE at the center of the channel, and the production of dissipation \((C_{\varepsilon 1})\) due to mean velocity gradients was varied to assess its influence on the predictions. If \( C_{\varepsilon 1} \) is set to 1.6 \((C_{\varepsilon p}=0.023)\), the TKE is matched near the wall and at the center of the channel. However, the results show that there is still a constant TKE near the center of the channel, and the TKE predicted by the model deviates slightly from the data between \(0.4 < y/h < 0.9\). The normalized velocity profile is shown in Fig. 15.

The predictions of the turbulent dissipation model with an additional production term due to the presence of particles are shown to improve the prediction of TKE and to also predict the turbulence modulation due to particle loading. Comparing Figs. 12 and 14 shows that the production of dissipation due to the particle surfaces mildly affects the TKE near the wall, and obviously the production of dissipation due to mean velocity gradients has no affect at the center of the channel. These results also highlight that \( C_{\varepsilon 1} \) and \( C_{\varepsilon 3} \) may be slightly coupled in regions where the velocity gradient is nonzero. Overall, the additional production term shows to be an improvement over the standard single phase turbulent dissipation model. For flows with low particle Reynolds numbers, the prescribed coefficients are invalid, and additional studies are needed to validate these conditions. More results with supporting data, such as the laden pressure gradient, wall shear, particle concentration, and velocity distributions, are needed to further validate the model.

5 Conclusion

A volume averaged equation set for particle-laden turbulent flow is presented. This includes momentum, TKE, and dissipation transport equations, coefficients, and boundary conditions for \( k \) and \( \varepsilon \). A numerical model was developed and compared with the continuous phase TKE and velocity data of Kulick et al. [8] and Paris and Eaton [6]. The model shows good agreement with the TKE and continuous phase velocity data. The model also predicts the attenuation of TKE for increased particle loading and compares reasonably well with the experimental data.

In this study, the coefficients associated with the production of dissipation were varied to appreciate their effect on model predictions. It was found that the coefficient for production of dissipation due to mean velocity \((C_{\varepsilon 1})\) gradients needs to be calibrated to balance the production of dissipation coefficient due to particle surfaces \((C_{\varepsilon 3})\). This study also showed that \( C_{\varepsilon 3} \) is not calibrated correctly for low particle Reynolds number flows, and additional data are needed to understand the effect of particle loading at low particle Reynolds numbers. Such data would provide better calibration of these coefficients.

References


