Numerical and Experimental Investigation of Performance Characteristics of Lithium/Sulfur Cells

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Abstract

In this study, we propose a modified mathematical model for Lithium/Sulfur (Li/S) cells and present a detailed numerical analysis showing the effects of (1) the presence of solid sulfur species (Li$_2$S$_{12}$, Li$_2$S$_{6}$, Li$_2$S$_3$), (2) the solubility and diffusivity of polysulfides in the electrolyte, and (3) reaction rate constants of polysulfide reduction reactions at Li electrode on the electrochemical performance characteristics of Li/S cells including the cycling performance. The cell potential profiles predicted from numerical analysis were compared with experimental discharge and charge curves with emphasis on the possibility of the presence of the intermediate solid phase Li$_2$S$_{12}$. Numerical results suggest that the cell potential obtained without the consideration of the intermediate solid phase Li$_2$S$_{12}$ is in the best agreement with experimental results. Also, the polysulfide shuttling phenomenon has been numerically analyzed with electrolytes of different polysulfide solubility and compared with experimental cell discharge and charge curves. Our model clearly shows that the electrolyte of high polysulfide solubility and unprotected negative (anode) electrode can lead to unwanted reduction reactions of high-order polysulfides on the Li electrode, resulting in a significant overcharge problem. This agrees well with our experimental results for identical experimental conditions. Furthermore, the cycling performance of a Li/S cell was predicted, including the effect of the polysulfide shuttle on the subsequent discharge/charge curves. In the simulation, it was found that the high voltage plateau is reduced markedly in the discharge process after incomplete charge because sulfur ($S_8$) could not be recovered when the polysulfide shuttle problem is significant. Also, it is suggested that cycling performance of Li/S cells would be improved by limiting both the polysulfide diffusivity and the reduction reaction rate of high-order polysulfides at Li electrode during charge.

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1. Introduction

Current lithium-ion (Li-ion) cells are approaching to their maximum specific energy (~200 Wh/kg at the cell-level) and are unable to provide sufficient energy storage capability for emerging technologies such as electric vehicles. Lithium/Sulfur (Li/S) cells can offer significantly greater practical (obtainable) specific energies (~600 Wh/kg), and can potentially lead to transformative changes in battery technology and replace current Li-ion cells [1, 2]. In addition to the high theoretical specific energy, Li/S cells have several other advantages. For instance, one of the primary constituents of the cell, sulfur, is inexpensive, abundant on earth and environmentally benign. However, the successful development of Li/S cells as a commercial product has been hindered by the poor cycling performance of these cells, caused largely by the dissolution of sulfur (loss as polysulfides) and movement of these polysulfide species back and forth between anode and cathode in commonly used liquid electrolytes. Therefore, one of the biggest challenges is the selection of an appropriate electrolyte that allows good cyclability and high specific energy without compromising on safety standards. In order to address this challenge, it is essential to unveil the reaction mechanisms such as reduction/oxidation reactions and their pathways on the active material as well as precipitation and dissolution phenomena of solid phases in the Li/S cell.

Extensive experimental studies have been carried out to elucidate the electrochemical reaction process during the operation of Li/S cells. Nelson et al. [3] used in-operando x-ray diffraction...
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description (Unit)</th>
</tr>
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<tbody>
<tr>
<td>( a )</td>
<td>Specific surface area of cathode (m²/m³)</td>
</tr>
<tr>
<td>( C_i )</td>
<td>Concentration of species ( i ) (mole/m³)</td>
</tr>
<tr>
<td>( C_{i,\text{ref}} )</td>
<td>Reference concentration of species ( i ) (mole/m³)</td>
</tr>
<tr>
<td>( C_{i,\text{ave}} )</td>
<td>Average concentration of species ( i ) in cathode (mole/m³)</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Diffusivity of species ( i ) (m²/s)</td>
</tr>
<tr>
<td>( F )</td>
<td>Faraday constant (C/mole)</td>
</tr>
<tr>
<td>( i_i )</td>
<td>Current density due to reaction ( i ) (A/m²)</td>
</tr>
<tr>
<td>( i_{i,\text{ref}} )</td>
<td>Exchange current density of electrochemical reaction (A/m²)</td>
</tr>
<tr>
<td>( i_{\text{load}} )</td>
<td>Load current density (A/m²)</td>
</tr>
<tr>
<td>( K_k )</td>
<td>Solubility parameter of precipitate ( k ) (mole/m³) (for ( S_{8(s)} ) mole³/m³ (for ( Li_2S_6(s) ) and ( Li_2S_2(s) )))</td>
</tr>
<tr>
<td>( k_{k,i} )</td>
<td>Rate constant of precipitate ( k ) (1/sec (for ( S_{8(s)} ) m⁶/mol²·sec (for ( Li_2S_6(s) ) and ( Li_2S_2(s) )))</td>
</tr>
<tr>
<td>( k_{ps} )</td>
<td>Reaction rate constant of high-order polysulfides reduction on Li anode (1/sec)</td>
</tr>
<tr>
<td>( N_f )</td>
<td>Flux of species ( i ) (mole/m²·sec)</td>
</tr>
<tr>
<td>( R )</td>
<td>Gas constant (/mole·K)</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>( t )</td>
<td>Time (sec)</td>
</tr>
<tr>
<td>( V_k )</td>
<td>Molar volume of the precipitate ( k ) (m³/mole)</td>
</tr>
<tr>
<td>( z_i )</td>
<td>Charge number of species ( i )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Symmetry coefficient</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Porosity of material</td>
</tr>
<tr>
<td>( \varepsilon_k )</td>
<td>Volume fraction of precipitate ( k )</td>
</tr>
<tr>
<td>( \phi_l )</td>
<td>Liquid phase electric potential (V)</td>
</tr>
<tr>
<td>( \phi_s )</td>
<td>Solid phase electric potential (V)</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>Surface overpotential for electrochemical reaction ( i ) (V)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Ionic conductivity (S/m)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Electric conductivity (S/m)</td>
</tr>
</tbody>
</table>

(XRD) to show structural and morphological changes as well as to identify polysulfide species involved in the electrochemical reactions of Li/S cells. Cuisinier et al. [4] used in-operando x-ray absorption spectroscopy (XAS) to study the mechanisms of sulfur redox chemistry during cycling, which showed how the sulfur fraction and sulfide precipitation impact capacity. However, those studies had the difficulty of identifying all polysulfide species during cell operation due to the limited resolution of in-operando XAS. Kawase et al. [5] analytically studied the electrochemical reaction process occurring within the cell using an organic conversion technique. In that work, ex-situ liquid chromatography-mass spectrometry (LC/MS) analysis was used to identify polysulfide species. Even though these studies clarified which polysulfide species are involved at certain discharge-charge depth, the precipitation/dissolution phenomena of lithium-polysulfides remain unclear.

A numerical analysis can be used to identify all reactant and product species involved in cell reactions during the discharge-charge process, including lithium ions, polysulfide ions and solid phase sulfur species. Therefore, one can investigate precisely the electrochemical behavior of a Li/S cell by performing numerical analysis based on a rigorous model. Kumaresan et al. [6] originally proposed a mathematical model for a Li/S cell to study the discharge process. Recently, Ghaznavi [7–9] expanded the model for sensitivity analysis by varying key parameters. However, those studies cannot be used to predict cycling performance of the cell since the model is missing an important reduction reaction of the high-order polysulfides (Sn²⁺, n ≥ 4) on Li electrode, which results in overcharge [10] and the polysulfide shuttling [11,12] problem. Earlier, Mikhaylik and Akridge [13] proposed a model describing reduction reactions of high-order polysulfides on Li electrode and discussed the shuttling problem, but their model did not include ion transport phenomena which would introduce concentration polarization. Recently, a mechanistic model including ion transportation was proposed by Hofmann et al. [14] to show the polysulfide shuttle and capacity loss in Li/S cells. However, the relation between polysulfide solubility and the shuttle problem is still unclear. Furthermore, the existence of the Li₂S₂(2+) phase near the end of the discharge process has been debated. For instance, in-operando XRD showed no formation of crystalline Li₂S [3], which contradicts ex-situ studies previously reported [15–17], while other in-situ studies indicated Li₂S₃(3) as a final stage crystalline lithium sulfide [4,18].

Therefore, in this work, we propose a modified mathematical model based on the work of Kumaresan et al. [6] and compare numerical results with experimental data to identify the ionic and solid phase species that are mainly involved in the electrochemical reactions in the Li/S cell. We use the experimentally validated numerical model to explain the roles of solid phase sulfur species for the electrochemical performance behavior of the Li/S cell. Moreover, the reduction reaction at the negative electrode during charge is modeled, and charging behavior is studied to reveal the polysulfide shuttling phenomenon leading to incomplete charge. Lastly, the cycling performances of the Li/S cell is analyzed with emphasis on the solubility and diffusivity of polysulfides and compared with experimental results of Li/S cells constructed with and without LiNO₃ additive in the electrolyte.

### 2. Theory

#### 2.1. Chemical Reactions

In this section, we briefly review the general chemical reactions for a Li/S cell. A Li/S cell consists of a lithium anode (negative electrode) and a sulfur-carbon cathode (positive electrode) as a starting configuration. Upon discharging, at the anode side, lithium oxidizes to form lithium ions in the electrolyte that moves to the cathode material through the separator/electrolyte, while at the cathode side sulfur is reduced first to higher-order polysulfides, and then to lower order polysulfides and, eventually, lithium ions and lower-order polysulfides react to produce lithium sulfide (Li₂S₂(s) and/or Li₂S₃(s)) during the discharge process and the reverse occurs during charge.

Fig. 1 shows the schematic structure of a Li/S cell and details the chemical reactions involved. Here, ions and solvated species involved in the electrochemical reactions are specified as Li⁺, Sn²⁻ (n = 1, 2, 4, 6, 8) and S₈(s). Chemical reactions for these species are described by the Butler-Volmer equation and will be discussed in the mathematical model section. Other components, such as Sn(1) Li₂S₇(s) and Li₂S₃(s) are regarded as possible solid phases and a precipitation/dissolution model is applied only for them. The oxidation (during discharge) and reduction (during charge) reactions of polysulfides can also occur at Li electrode since polysulfide dissolves into the electrolyte and diffuses to the Li electrode side. However, the reduction reaction (e.g.: \( S_{n(s)} − \rightarrow Sn^{2−} \)) is considered at the negative electrode side during only the charge process, since the reduction of high-order polysulfides on the Li anode is only significant during the charging process [10,19].

#### 2.2. Modified Mathematical Model

Although Kumaresan et al. [6] have presented the basic mathematical model for the Li/S cell, their proposed model is
numerically unstable for high solubility of lithium polysulfides. For instance, the volume fractions ($e$) of Li$_2$S$_{2(3)}$ and Li$_2$S$_{4(5)}$ could approach zero during the calculations of higher-potential discharge due to the higher solubility of lithium polysulfides. Once the volume fraction becomes zero, the precipitation to Li$_2$S$_{2(3)}$ and Li$_2$S$_{4(5)}$ cannot occur in the positive electrode side during discharge even if the concentration of S$^2^-$ or S$^-$ is over the solubility limit. Also, the volume fraction of $S_{(0)}$ can suffer from the same numerical instability during the charge process. Thus, to circumvent the undesired numerical issue, the precipitation and dissolution model is modified in our numerical scheme by adding an extra term as

\[
\frac{dG_{S_{(0)}}}{dt} = V_{S_{(0)}} \left[ k_{S_{(0)}} e_{S_{(0)}} \left( C_{S_{(0)}} - K_{S_{(0)}} \right) + k_{S_{(0)}} \frac{C_{S_{(0)}}}{c_{S_{(0)}}} \right]
\]

(1 - a)

\[
\frac{dG_{Li_{2}S_{2(3)}}}{dt} = V_{Li_{2}S_{2(3)}} \left[ k_{Li_{2}S_{2(3)}} e_{Li_{2}S_{2(3)}} \left( C_{Li_{2}S_{2(3)}}^2 C_{S_{(0)}}^2 - K_{Li_{2}S_{2(3)}}^2 \right) + k_{Li_{2}S_{2(3)}} \frac{C_{Li_{2}S_{2(3)}}^2}{G_{Li_{2}S_{2(3)}}} \right]
\]

(1 - b)

\[
\frac{dG_{Li_{2}S_{4(5)}}}{dt} = V_{Li_{2}S_{4(5)}} \left[ k_{Li_{2}S_{4(5)}} e_{Li_{2}S_{4(5)}} \left( C_{Li_{2}S_{4(5)}}^2 C_{S_{(0)}}^2 - K_{Li_{2}S_{4(5)}}^2 \right) + k_{Li_{2}S_{4(5)}} \frac{C_{Li_{2}S_{4(5)}}^2}{G_{Li_{2}S_{4(5)}}} \right]
\]

(1 - c)

where $V$, $k$ and $K$ are molar volume, precipitation/dissolution rate constant, and solubility parameter, respectively. In these equations, the additional term is only effective if the value of the volume fraction is very small.

Moreover, in our study, a reduction reaction of high-order polysulfides at the negative electrode is added to investigate the polysulfide shuttling problem. The rate of the high-order polysulfide reduction reaction at the negative electrode surface is directly related to the concentration of high-order solvated polysulfides as

\[
\frac{dC_{S_{(0)}}}{dt} = - \left( k_{ps1} C_{S_{(0)}} \right)
\]

(2 - a)

\[
\frac{dC_{S_{(0)}}}{dt} = \left( -k_{ps2} C_{S_{(0)}}^2 + k_{ps1} C_{S_{(0)}} \right)
\]

(2 - b)

\[
\frac{dC_{S_{2(3)}}}{dt} = \left( -k_{ps3} C_{S_{2(3)}} - 4 k_{ps2} C_{S_{(0)}} \right)
\]

(2 - c)

\[
\frac{dC_{S_{4(5)}}}{dt} = \left( -k_{ps4} C_{S_{4(5)}} + 3 k_{ps3} C_{S_{2(3)}} \right)
\]

(2 - d)

\[
\frac{dC_{S_{6(7)}}}{dt} = \left( -k_{ps5} C_{S_{6(7)}} + 2 k_{ps4} C_{S_{4(5)}} \right)
\]

(2 - e)

where $k_{ps}$ is the reaction rate constant of high-order polysulfides reduction. As mentioned in the previous section, it is assumed that the high-order polysulfides reduction reaction at Li electrode occurs during the charge process only. All other governing equations and boundary conditions are introduced in Tables 1 and 2, respectively.

2.3. Computational Method

2.3.1. Assumptions

Dendrite formation and sulfur agglomeration are not considered in this macroscopic model. Thus, a one-dimensional analysis is sufficient to show the battery performance. Other assumptions used in this study are

1. Metallic type lithium foil is used as anode material so that porous separator and cathode are only considered as computational domain.
2. Species concentration gradient is dominant from anode to cathode (or cathode to anode) since the species transport occurs mainly in that direction.
3. Effect of EDL (electrode double layer), SEI (solid-electrolyte interface) and dendrite formation are negligible for capturing macroscopic phenomena.
Table 1

Chemical reactions at the anode and cathode, and mathematical models for species concentrations and electric potentials. Properties and constant values are same as reported in [6].

<table>
<thead>
<tr>
<th>Anode reaction</th>
<th>Cathode reaction</th>
<th>Electric potential in electrolyte</th>
<th>Electric potential in cathode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Li \rightarrow Li^+ + e^-$</td>
<td>$1/2S_0^2 \rightarrow \frac{1}{2}S_0^2$</td>
<td>$\nabla \cdot (\lambda \nabla \phi) = -\sigma (i_1 + i_2 + i_3 + i_4 + i_5)$</td>
<td>$\nabla \phi = 0$</td>
</tr>
<tr>
<td>$2S_0^2 \rightarrow e^- + 2S_0^2$</td>
<td>$2S_2^- \rightarrow e^- + S_2^2$</td>
<td>$\nabla \cdot (\lambda \nabla \phi) = -\sigma (i_1 + i_2 + i_3 + i_4 + i_5)$</td>
<td>$\sigma \nabla \phi = -i_{load}$</td>
</tr>
</tbody>
</table>

4. The high-order polysulfide reduction reactions at Li electrode occur during the charge process only.
5. Diffusion correlation between each species is weak so that self-diffusion coefficient is used for species transport equations.

3. Experimental Section

Sulfur-graphene oxide (S-GO) nanocomposite powders were synthesized using a method previously reported [1,22] and used as active material for the sulfur electrodes in this study. The sulfur electrodes were fabricated by mixing the S-GO nanocomposite, carbon black (Super C65) with a binder, polyvinylidene fluoride (PVDF), at a weight ratio of 70:20:10 in N-methyl-2-pyrrolidone (NMP) solvent to form slurry using an ultrasonicator. The resulting slurry was deposited on pure aluminum foil using a doctor blade (Elcometer 3540 Bird Film Applicator). After drying in a vacuum oven at 50 °C for 48 hours, the electrode was cut into circular pieces with a diameter of 12.7 mm for L/S cell assembly.

For the electrolyte, 1 M lithium bis(trifluoromethylsulfonyl) imide (LiTFSI, Sigma-Aldrich) in an organic solvent mixture of 1,3-dioxolane (DOL) and 1,2-dimethoxyethane (DME) mixture (1:1 by volume) was prepared and used for evaluation of the electrochemical performance of the Li/S cell with electrolytes of high polysulfide solubility [23]. For electrolytes of low polysulfide

2.3.2. Numerical Scheme

We used a finite volume method to construct linear algebraic equations for numerical modeling of the Li/S cell [20,21]. Discretized algebraic equations were obtained using a power law scheme for both mass conservation and charge conservation equations. Time dependent equations were solved by an implicit method for numerical stability. The Thomas algorithm was used to solve the tri-diagonal system of linear algebraic equations. The tolerances for the mass and charge conservation equations are $10^{-5}$. Numerical simulations were performed on an Intel Xeon 2.2GHz CPU.
Fig. 2. (a) Numerically predicted and experimentally observed cell potential profiles during the discharge process for a current density of 0.1C. A hybrid electrolyte consisting of ionic liquid and organic solvent (1 M LiTFSI in PYR14TFSI/DOL/DME mixture (2:1:1 by volume)) was used as electrolyte in the experimental work. Four different cases are considered in the numerical study based on the different proportions of crystalline Li$_2$S$_2$(s) and Li$_2$S(s). Numerically predicted volume fractions of crystalline Li$_2$S$_2$(s) and Li$_2$S(s) for (b) case 1, (c) case 2, (d) case 3, and (e) case 4. Volume fraction values are averaged in the sulfur electrode region [6].
solubility, n-methyl-(n-butyl) pyrrolidinium bis(trifluoromethane-
sulfonyl)imide (PYR14TFSI, Sigma-Aldrich) was introduced to form
1 M LiTFSI in PYR14TFSI/DOL/DME mixture (2:1:1 by volume) as a
pyrrolidinium-based ionic liquid. PYR14TFSI has been used
successfully as an effective solvent to minimize the polysulfide
shuttle with lower polysulfide solubility [24,25].

CR2032-type coin cells were assembled by sandwiching two
separators (Celgard 2400) between a lithium metal foil (99.98%,
Cyprus Foote Mineral) and a sulfur electrode fabricated with the S-
GO composite in an argon-filled glove box. Constant-current
discharge and charge testing of the coin cells were performed using
a batteryycler (Maccor System 4000) at a load current density of
0.1C (1C = 1675 mA/g) between 1.7 and 2.8 V. The cell capacity was
normalized by the weight of sulfur. All electrochemical
characterizations were conducted inside a temperature chamber (TestEquity
TEC1) controlled at 30 °C.

4. Results and Discussion

4.1. Discharge and Charge Characteristics

In a Li/S cell, it has been reported that two plateaus appear in
the cell potential profile during the discharge process. The first
(high voltage) plateau requires constant concentration of S0(s) for
production of S8(l), which is the significant reduction reaction.
The constant concentration of S0(s) can be maintained if S0(s) can be
continuously dissolved into the electrolyte. On the other hand, the
second plateau is due to the formation of solid lithium sulfides
(Li2S2(s) and/or Li2S3(s)) while it is not clear which crystalline species
is formed or dominant during the discharge process. For instance,
Nelson et al. claimed “it was found that crystalline Li2S does not form
at the end of discharge for all sulfur cathodes studied” [3], but
Cuisinier et al. reported crystalline Li2S as the discharge product at
the end of discharge [4]. On the other hand, Manthiram et al.
suggested Li2S2(s) and Li2S3(s) as the final discharge product [26].
Therefore, in this study, we investigated the cell potential profiles
depending on the relative presence of solid product species Li2S2(s)
and Li2S3(s) to show the possible final solid phase in a Li/S cell. The
numerical simulation has been carried out for four different
conditions. In Case 1, it was hypothesized that crystalline Li2S2(s)
rarely forms, while, in Case 4, the crystalline Li2S2 formation is
significant. Case 2 and Case 3 assumed that the crystalline
formation of Li2S2 is more than Case 1 and less than Case 4,
respectively.

Fig. 2(a) shows the cell potential profiles for each case. It is
notable that, for the all cases, a small valley appears near at the 30%
depth of discharge (DOD), in which precipitation started. The
precipitation reaction mitigates saturation of polysulfide
(S2−n− and S2−n) [6] and it alleviates a potential drop due to
concentration polarization. After this singular point (over 30%
DOD), each case produces a different cell potential profile. In Case
1, the simulation shows the most linear cell potential profile and
full discharge depth is obtained. Here, specific capacity at 100% DOD
is 1675 mAh/g and the value is the same as the theoretical
specific capacity which can be achieved in a Li/S cell. However, the
cell potential profiles were distorted in Cases 2 and 3 and, eventually,
the cell potential sharply dropped before reaching full discharge in Case 4. In Cases 2 and 3, a higher cell potential appears in the 30 ~ 55% DOD region since supersaturated S2−n− precipitates
to Li2S2(s) (see Fig. 2(c) and (d)), and it alleviates the potential loss
due to concentration polarization. However, the cell potential
drops to a lower value after dissolution of Li2S2(s) starts (over 55%
DOD). Here, the lower potential is obtained due to the kinetic
resistance of the solid phase change, Li2S2(s) → Li2S3(s) (see Fig. 2(c)
and (d)). For Case 4, the precipitation of Li2S2(s) is significant and
mostly due to S2−n− ions, reduced from high-order polysulfide,
precipitating to Li2S2(s) (see Fig. 2(e)). For this case, the discharge
potential suddenly drops at 55% DOD when the fraction of Li2S2(s)
reaches its maximum value. At this stage, the discharge reaction
solely depends on the solid-state phase change reaction (Li2S2
(s) → Li2S3(s)) and this solid state reaction introduces large kinetic
resistance, resulting in a massive potential drop [27,28].

As shown in Fig. 2 (a), the cell potential profile significantly
varies with the presence of Li2S2(s) and, by comparing cell potential
profiles between numerical analysis and experimental results, it
can be suggested which model (presence or absence of Li2S2(s)) can
properly predict the Li/S cell performance during the discharge
process. In Fig. 2 (a), the circle symbols show the cell potential
profile obtained experimentally. The experimental profile clearly
demonstrates the appearance of two plateaus and especially,
distortion of the second plateau cannot be seen. Here, the cell
potential profile obtained from the numerical analysis for Case 1 is
the only model in good agreement with the experimental results.
Hence, the absence of Li2S2(s) during the discharge process (Case 1)
is considered as the most reasonable model for this system.

Fig. 3. (a) Numerical and experimental cell potential profiles during the discharge and charge processes for a current density of 0.1C. A hybrid electrolyte consisting of ionic liquid and organic solvent (1 M LiTFSI in PYR14TFSI/DOL/DME mixture (2:1:1 by volume)) was used in the experimental study. (b) Volume fractions of crystalline S8(l), and Li2S2(s) during the discharge and charge processes.
Moreover, there are several references [4,29] to support this assumption. Referring to the comparison between numerical and experimental results as well as the literature, we modified the mathematical model by considering $S_{8(s)}$ and $Li_2S_{8(s)}$ as the only possible solid species in the Li/S cell and, used this modified model in the rest of this paper.

In addition to the cell performance during discharge, the cell potentials are calculated at different depths of charge (DOC) and compared with the experimental results (Fig. 3). Similar to the discharge process, the cell potential strongly depends on the dissolution of $Li_2S_{8(s)}$ and precipitation of crystalline $S_{8(s)}$. The charge potential gradually increases with DOC as shown in Fig. 3(a). As aforementioned in the analysis of the discharging process, the concentration of reactant ($S^2$) has been maintained in 0 ~ 80% for the DOC region during the charge process because of the continuous dissolution of $Li_2S_{8(s)}$. Therefore, the potential drop due to concentration polarization is insignificant at this stage. However, once $Li_2S_{8(s)}$ is completely dissolved and depleted (see Fig. 3(a) and (b), at 80% DOC), the charge potential increases more due to the concentration polarization of the reactant. After that stage, the charge potential increase is slightly alleviated due to precipitation of $S_{8(s)}$, resulting in reduced $S_{8(s)}$ concentration, but, eventually, the potential soars sharply near the fully charged stage. Here, the predicted charge profile is compared with the experimental results. In Fig. 3 (a), delta symbols show the cell potential profile obtained from experiments during the charge process. In the beginning of the charging stage, a large potential barrier is observed. After the barrier, the charge potential increases gradually and it is elevated more near the 80% DOC. However, the potential increase is alleviated soon and, eventually, it soars near the fully charged state (100% DOC). Here, the potential barrier at the beginning of the charge process is due to the high electronic resistivity and low lithium ion diffusivity in $Li_2S_{8(s)}$ [30]. In our continuum model, the microscopic mechanism is not involved and the simulation could not capture this potential barrier. However, except for the barrier, the experimental charge potential profile is in good agreement with our numerical results (see Fig. 3 (a)). By comparing the numerical and experimental results, we adequately validated the modified numerical model that can precisely elucidate the discharge and charge processes in Li/S cells.

4.2. Modeling of Overcharge Problem and Implications

In Li/S cells, the reduction reaction of high-order polysulfides at the negative electrode introduces polysulfide shuttling resulting in a cell overcharge problem. As mentioned in the chemical reaction section, during the charge process, $S_{8(s)}$, oxidized from $S_{8}^{2-}$, moves to the negative electrode and is reduced to $S_{8}^{2-}$ (see Fig. 1). Similarly, $S_{8}^{2-}$, $S_{6}^{2-}$, $S_{4}^{2-}$, $S_{2}^{2-}$ and $S^{2-}$ can be reduced to low-order polysulfides and lithium sulfide. Therefore, the charging current is partly used to reduce high-order polysulfides at the negative electrode in addition to the regular oxidation reaction at the positive electrode for formation of polysulfides during charge process. This is commonly known as the overcharge problem. In this study, to reveal the overcharge phenomena in the Li/S cell, the charge behavior was investigated with two extreme cases. In the first case, polysulfide shuttling is set to be insignificant due to the low solubility of polysulfides and the slow reduction reaction for polysulfides at the negative electrode. In the other case, the shuttling can be significant because of high solubility and a fast reduction reaction.

Here, the solubility of polysulfides can be controlled with the values of the solubility parameter ($K_s$), with greater $K_s$ values indicating higher solubility of polysulfides. The solubility parameter values that are used in the simulation for each case are shown in Table 3. Solubility parameters were calculated from maximum concentration of species that can be dissolved into an electrolyte. For example, $K_{S_{8(s)}}$ value is obtained directly from maximum concentration of dissolved $S_8$ in an electrolyte. The precipitation of solid $S_8$ occurs when concentration of $S_{8(s)}$ is higher than the solubility parameter value and vice versa for dissolution. Similarly, value for $K_{Li_2S_{8(s)}}$ was obtained from maximum soluble concentration of $Li_2S$ as $K_{Li_2S_{8(s)}} = C_{Li_2S_{8(s)}}^{max}$. Here $C_{Li_2S}$ is the initial concentration of lithium ion in the electrolyte.

The reduction reaction rate can be controlled with the reduction reaction rate constant ($K_{RC}$). For the low polysulfide solubility and slow reduction reaction (Case-A, Table 4), the Li/S cell could be fully charged as shown in Fig. 4(a). Here, the concentration of $S_{8(s)}$ is under 10 mole/m³ as shown in Fig. 4(b) and $S_{8}^{2-}$ is fully converted to $S_{8(s)}$ at the end of the charge (see Fig. 4(c)). At the same time, $S_{8(s)}$ is also precipitated as $S_{8(s)}$ and the volume fraction of $S_{8(s)}$ has approximately reached to its initial value as shown in Fig. 4(c). The polysulfide oxidation reaction happens progressively from $S_{8}^{2-}$ to $S_{8(s)}$ and, eventually, $S_{8(s)}$ is fully recovered since the reduction reaction of polysulfides is weak at the negative electrode.

The numerically predicted cell performance characteristics are consistent with our experimental results. In Fig. 4(a), circle symbols show experimental cell potential for the Li/S cell using the electrolyte (1 M LiTFSI in P14TFSI/DOL/DME mixture) with LiNO₃ additive. In this case, the cell can be fully charged in the presence of LiNO₃ because the anode protection layer (APL) can be formed at Li electrode surface to block the reduction reaction by adding LiNO₃ to the electrolyte. Moreover, in this case, since the ionic liquid-based electrolyte which has low polysulfide solubility [31] is used as the electrolyte, the high-order polysulfide reduction reaction is insignificant so that the overcharge problem is rarely noticed. This result agrees well with our simulation results that use the condition of low polysulfide solubility and low reaction rate constant of high-order polysulfide reduction (see Fig. 4(a)).

In contrast, Fig. 5(a) shows numerical results with high solubility for polysulfides and considerable reduction reaction at the negative electrode surface. The reduction reaction constants for each polysulfide specified in Table 4 were used to study the reaction pathways resulting in incomplete charging. Serious overcharge problems are observed for all simulation cases. Constant charging potential was noted after a certain depth of charge stage, and the cell could not reach the cutoff voltage corresponding to the fully charged condition.

The constant charging potentials vary considerably in each case. For example, the charging value is limited at 2.35 V for the Case-B which illustrate that the reduction reaction of $S_{8(s)}$ is most

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#### Table 3

<table>
<thead>
<tr>
<th>Simulation conditions</th>
<th>Solubility parameters</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$K_{S_{8(s)}}$</td>
</tr>
<tr>
<td>Low solubility of polysulfide</td>
<td>10 mole/m³</td>
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<tr>
<td>High solubility of polysulfide</td>
<td>150 mole/m³</td>
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#### Table 4

<table>
<thead>
<tr>
<th>Simulation Case</th>
<th>Reaction rate constant (1/sec)</th>
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<tbody>
<tr>
<td></td>
<td>$k_{RC1}$</td>
</tr>
<tr>
<td>Case-A</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>Case-B</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>Case-C</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>Case-D</td>
<td>$1 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
significant. Meanwhile, lower constant potentials, 2.32 V and 2.28 V, are observed for the Case-C and D, respectively. In Case-B, even though $S_{8(l)}$ can be recovered with oxidation reaction from $S_{8}^{2-}$ at the positive electrode, the concentration of $S_{8(l)}$ is immediately reduced due to the formation of $S_{8}^{2-}$ by the reduction reaction at the negative electrode at the same time (see Fig. 5(b)). Therefore, the charging potential is maintained at the reaction potential due to incomplete polysulfide recovery and infinitely long charging occurs. However, in Case-C, it is more difficult to recover $S_{8(l)}$ from $S_{8}^{2-}$ (see Fig. 5(c)) since most of the $S_{8}^{2-}$ intensively reduces to $S_{6}^{2-}$ at the negative electrode before $S_{8}^{2-}$ oxidizes to $S_{8(l)}$ at the positive electrode. As a result of incomplete recovery, the charging potential is limited by the oxidation reaction of $S_{8}^{2-}$. For case D, less $S_{8(l)}$ and $S_{8}^{2-}$ are recovered than in Case-B and C (see Fig. 5(d)), and the lower charging potential is observed.

Considering the charging behaviors, it is noteworthy that the charging potential significantly varies with the reduction reaction pathway at the negative electrode. Hence, we can find which specific reaction at the negative electrode can lead to the infinite charging problem by comparing simulation and experimental cell potential profiles. For the comparison, the Li/S cell was tested with an inferior electrolyte and electrode, which are likely to induce a significant polysulfide shuttling problem. 1 M LiTFSI in DOL/DME without LiNO$_3$ was used as electrolyte with high solubility of polysulfides because the organic electrolyte has a high solubility for polysulfide compared to ionic liquids. Furthermore, the polysulfide reduction reaction is significant at the negative electrode during charging because of the absence of the APL (no LiNO$_3$). In this experiment, the infinite charging problem was also observed (see Fig. 5(a) circle symbols). The experimental charging behavior of the cell follows the trend predicted by the model with a significant reduction reaction of $S_{8}^{2-}$ (simulation Case-D). Therefore, we can attribute the overcharge problem to the significant reduction reaction at the negative electrode and high solubility of polysulfides in the electrolyte. The results also suggest that $S_{8}^{2-}$ reduction mainly led to the infinite charging phenomenon in this system.

4.3. Prediction of Cycling Performance

Cycling performance is a key issue in Li/S cells. It is well known that Li/S cells suffer from short cycle lives due to the polysulfide shuttling. As mentioned in the previous section, the overcharge problem is significant when the high-order polysulfide reduction reaction on the Li electrode occurs intensively. Hence, in this section, cycling performance is predicted and investigated with
Fig. 5. (a) Numerical and experimental charge curves of a Li/S cell. An organic electrolyte with high polysulfide solubility (1 M LiTFSI in DOL/DME mixture (1:1 by volume)) was used in the experimental work. The reaction rate constants of high-order polysulfides are given in Table 4 for case- B, C and D. Numerically predicted (averaged) concentration of the solvated polysulfide for (b) Case-B (c) Case-C (d) Case-D in the cathode region during the charge process for a Li/S cell modeled using an electrolyte with high polysulfide solubility.

Fig. 6. Simulated cell potential profile for the 1st and 2nd discharge and 1st charge processes for varying reduction reaction rate constant of high-order polysulfide on the Li anode. For case (a) insignificant reduction reaction (b) intensive reduction reaction at anode side during charge state.
changes in intensity of the reduction reaction (Case A and D). Second discharge profiles were obtained for Case A and Case D as shown in Fig. 6. For Case A, the characteristic first and second plateaus are observed in the second discharge curve and the cell potential profile exactly corresponds to the first discharge profile (see Fig. 6 (a)). However, for Case D, the first potential plateau disappears in second discharge and the total specific capacity is also decreased (see Fig. 6(b)). As shown in the previous section, even if the cell was slightly overcharged for Case A, \( S_{8}^{2-} \) was fully recovered to \( S_{8}^{2-} \) so that the reduction reaction \( (S_{8}^{2-} \rightarrow S_{8}^{2-}) \) happens in the first plateau region (0~30% DOD) (see Fig. 7 (a)). However, for Case D, the \( S_{8}^{2-} \) concentration was constant during the overcharge process due to the polysulfides reduction reaction on the Li electrode and \( S_{8}^{2-} \) was recovered at less than 30 mM. Therefore, in this case, the reduction reaction \( (S_{8}^{2-} \rightarrow S_{8}^{2-}) \) of higher order polysulfide \( S_{8}^{2-} \) disappears during the second discharge due to lack of reactant and this results in a lower specific capacity (see Fig. 7 (b)). This poor cycling performance has been reported for a Li/S cell consisting of a non-protected Li electrode and an electrolyte which has high solubility of polysulfides [1,31–33].

To investigate effects of polysulfide diffusivity and APL for the Li/S cell cycling performance, the simulations were carried out with different diffusivity values of polysulfides (Table 5) and various reduction reaction rate constants of polysulfides at the negative electrode. First, it was assumed that the electrolyte has high diffusivity and the reduction reaction at the negative electrode is fast. In this case, the specific capacity faded rapidly at 920 mAh/g after only five cycles (Fig. 8(a)). However, by limiting the reduction reaction of polysulfides at the negative electrode, we could predict better cycling performance (Fig. 8(b)). In this case, the specific capacity was maintained at over 1200 mA/h after the fifth cycle of operation. In addition, the rapid capacity fading could be alleviated by limiting the polysulfide mobility (see Fig. 8(c)). Even though high order polysulfides could still be reduced and consumed at the negative electrode during charging, the low mobility of polysulfides resulted in a low local concentration of polysulfides at the negative electrode, which suppressed the reduction reaction and improved the cycling performance. However, in this case, the overcharge problem was still evident due to the fast reaction at the negative electrode. Obviously, the simulation results showed that the best performance can be obtained with a slow reduction reaction of polysulfides at Li electrode and low polysulfide diffusivity (Fig. 8(d)). In this last case, the overcharge problem was hardly observed and the simulation showed 1300 mAh/g specific capacity after the fifth cycle.

5. Conclusions

A modified model for the Li/S cell was proposed for analysis of the electrochemical characteristics of the Li/S cell. A reduction reaction of high-order polysulfides on the Li electrode was included in the model to investigate the polysulfide shuttling problem. Moreover, the existing kinetic model for the precipitation and dissolution was modified by adding an extra term for better numerical stability. The cell potential profiles were investigated considering the presence of solid sulfur phases \( Li_{2}S_{1} \) and \( Li_{2}S_{2} \). By comparing the numerical results with experimental data obtained with different electrolytes, it is suggested that a mathematical model considering \( S_{8}^{2-} \) and \( Li_{2}S_{2} \) as the only possible solid species can predict the electrochemical performance of the Li/S cell most accurately. Also, the charging behavior of Li/S cells was modeled and analyzed with variations of the reaction rate constant of polysulfide reduction on the Li electrode and compared with experimental results for Li/S cells assembled with electrolytes containing LiNO\(_{3}\) additive and ionic liquids. Numerical results reveal that the overcharge problem is significant with a higher reaction rate constant of polysulfide reduction on the Li electrode and cell charging is incomplete with electrolytes of high polysulfide solubility. These results agree well with experimental results of Li/S cells constructed with organic electrolytes in the absence of LiNO\(_{3}\). Moreover, it is noted that the high voltage plateau was reduced markedly in the discharge process after incomplete charge because sulfur \( S_{8} \) could not be recovered.

Table 5
Diffusion coefficients of Li ion and Polysulfides. Diffusivity values are assumed based on viscosity of electrolyte mixture.

<table>
<thead>
<tr>
<th>Species</th>
<th>( D_{i} (m^{2}/sec) )</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>High Diffusion Case</td>
</tr>
<tr>
<td>( Li^{+} )</td>
<td>5 \times 10^{-9}</td>
</tr>
<tr>
<td>( S_{8}^{2-} )</td>
<td>1 \times 10^{-8}</td>
</tr>
<tr>
<td>( S_{8}^{2-} )</td>
<td>2 \times 10^{-9}</td>
</tr>
<tr>
<td>( S_{8}^{2-} )</td>
<td>4 \times 10^{-9}</td>
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<td>6 \times 10^{-9}</td>
</tr>
<tr>
<td>( S_{8}^{2-} )</td>
<td>8 \times 10^{-9}</td>
</tr>
<tr>
<td>( S_{8}^{2-} )</td>
<td>1 \times 10^{-8}</td>
</tr>
</tbody>
</table>

Fig. 7. Concentration of \( S_{8}^{2-}, S_{8}^{2-} \) and \( S_{8}^{2-} \) in the cathode region for varying reduction reaction rate constant of high-order polysulfide on the Li anode during second discharge. For case (a) insignificant reduction reaction (b) intensive reduction reaction at the negative electrode during charging.
when the polysulfide shuttle problem is significant. Also, the cycling performance of the Li/S cell was investigated for various diffusivities of polysulfides and intensities of the reduction reaction at the negative electrode. Poor cycling performance was observed for high polysulfide diffusivity and intense reduction reaction at the negative electrode which resulted in fast fading of specific capacity. However, notable improvement of cycling performance was achieved by limiting the polysulfide diffusivity and the reduction reaction rate at the negative electrode during charge.

Acknowledgment

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References


